

On measurements of the energy and polarization distributions of high energy γ -beams

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Abstract

A possibility to measure the energy and polarization distributions of high intensity γ -beams is considered. This possibility is based on measurements of the number of e^\pm -pairs in such media, as laser waves and single crystals. The method may be useful for future $\gamma\gamma$ and $e\gamma$ -colliders.

1 Introduction

High energy γ -beams using in physical experiments have rather broad energy spectra. The measurement of the energy spectrum is an important and primary problem in such experiments. Often the intensity of γ -beams may be very high, and because of this it is impossible to use the number of γ -quanta counting techniques. An excellent example of this situation is the γ -beams on future $e\gamma$ and $\gamma\gamma$ -colliders[1, 2, 3, 4, 5]. In these cases for the correct interpretation of experimental data it will be important to know the spectral luminosity and its polarization characteristics.

In this paper we discuss a possibility to measure the spectral and polarization distributions of high intensity γ -beams with energies of hundreds GeV.

2 Main principles of measurement

Let us consider a parallel beam of high energy γ -quanta with some (unknown) energy distribution. We assume that this distribution is not changed for the time of measurements. In particular, this situation will take place on future $\gamma\gamma$ -colliders. Really, these $\gamma\gamma$ -colliders are accelerators with the periodic action when parameters of beams are maintained the same in each cycle.

It is known that γ -quanta are detected by their interactions with various media. The process of e^\pm -pair production followed by the development of electromagnetic cascade is the main result of such interaction. In this paper we consider a possibility to measure the energy and polarization distributions of high energy γ -beams with the use of thin layer of some specific medium, in which the e^\pm -pair production cross section is known with a high precision.

It is well known that the cross sections of e^\pm -pair production for high energy γ -quanta in amorphous media are practically independent of their energy [6]. However, there exist

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media, in which the cross sections of e^\pm -pair production are dependent on the energy and polarization of high energy γ -quanta. The monochromatic laser wave [7] and oriented single crystals [8, 9] are examples of such media (see Fig.1).

First we consider the possibility of measurements of the energy spectrum with the use of monochromatic laser wave. In this case the cross section of the e^\pm -pair production has the following form [10, 11, 12]:

$$\sigma_{\gamma\gamma}(z) = \sigma_0(z) + \sigma_c(z)P_2\xi_2 + \sigma_l(z)(\xi_1P_1 + \xi_3P_3), \quad (1)$$

where $z = \frac{m^2c^4}{E_\gamma E_l}$ is the invariant variable, E_γ and E_l are the energies of γ -quantum and laser photon, m is the electron mass, c is the speed of light, ξ_i and P_i , ($i = 1 - 3$) are the Stokes parameters of γ -quantum and laser photon, respectively. Functions σ_0 , σ_c , σ_l one can find in [10, 11, 12]. These three functions are different from zero at $0 < z \leq 1$. It means that the pair production in field of the laser wave is a threshold process. Note that the variable z is written for the counter motion of γ -quanta and laser wave. Generally, $z = 4m^2c^4/s$, where s is the total energy of the γ -quantum and the laser photon in their center-of-mass system.

In the general case of two colliding beams with corresponding densities (numbers of particles per volume unite) n_1 and n_2 , and velocities \mathbf{v}_1 , \mathbf{v}_2 the number of interactions in volume dV for a time dt is equal to [13]:

$$d\nu = \sigma \sqrt{(\mathbf{v}_1 - \mathbf{v}_2)^2 - [\mathbf{v}_1, \mathbf{v}_2]^2} n_1 n_2 dV dt, \quad (2)$$

where σ is the cross section of interaction. In the case when \mathbf{v}_1 and \mathbf{v}_2 are directed along the straight line this equation has the following form:

$$d\nu = \sigma |v_1 - v_2| n_1 n_2 dV dt \quad (3)$$

For $\gamma\gamma$ -interactions these equations are simplified:

$$d\nu = (1 - \cos \theta) \sigma n_1 n_2 dV dt, \quad (4)$$

$$d\nu = 2\sigma n_1 n_2 dV dt \quad (5)$$

where θ is the angle between directions of motion of these beams.

Let n_γ and n_l are the densities of high energy γ -beam and laser beam, correspondingly. In this case it would appear reasonable that $n_l \gg n_\gamma$. It means that we can neglect the variations of n_l -value in the considered process. Now we can find the total number of interactions in the thin volume V of laser bunch

$$n_i(E_l, P_1, P_2, P_3) = 2n_l \delta x \int_{E_{\gamma, \min}}^{E_{\gamma, \max}} N_\gamma(E_\gamma) \sigma_{\gamma\gamma}(E_\gamma, E_l) dE_\gamma, \quad (6)$$

where N_γ is the energy distribution of γ -beam, $E_{\gamma, \min}$ and $E_{\gamma, \max}$ are the minimum and maximum energies of γ -quanta, δx is the thickness of the laser bunch ($\delta x n_l \sigma_{\gamma\gamma} \ll 1$). It can be assumed that the photon energy in Eq. (6) is variable. It means that the laser frequency is changed for each or some cycles of $\gamma\gamma$ -collider. Besides, we propose that the number n_i can be measured with the help of some detector and it can be a scintillation counter. The number of photons creating from passing relativistic particles through the counter in a time of some picoseconds are proportional to the number of these particles. It may be another type of detector, for example, a current transformer, etc.

However, the cross section $\sigma_{\gamma\gamma}$ depends on the Stokes parameters of the γ -beam and laser wave. Because of this, one can duplicate the total number of measurements of n_i -value. One-half of measurements must be done with the circular polarization of laser wave $P_2 = 1$ and the remaining part - with the $P_2 = -1$. Thus we can get the following equations:

$$\begin{aligned} n_0(E_l) &= (n_i(E_l, 0, 1, 0) + n_i(E_l, 0, -1, 0))/2 = \\ &= 2n_l\delta x \int_{E_{\gamma,min}}^{E_{\gamma,max}} N_{\gamma}(E_{\gamma})\sigma_0(E_{\gamma}, E_l)dE_{\gamma}, \end{aligned} \quad (7)$$

$$\begin{aligned} n_c(E_l) &= (n_i(E_l, 0, 1, 0) - n_i(E_l, 0, -1, 0))/2 = \\ &= 2n_l\delta x \int_{E_{\gamma,min}}^{E_{\gamma,max}} N_{\gamma}(E_{\gamma}) <\xi_2(E_{\gamma})> \sigma_c(E_{\gamma}, E_l)dE_{\gamma}, \end{aligned} \quad (8)$$

where $<\xi_2(E_{\gamma})>$ is the circular polarization averaged over set of γ -quanta with the energy equal to E_{γ} . Eqs. (7)-(8) represent the system of two linear integral equations relative to functions $N_{\gamma}(E_{\gamma})$ and $N_{\gamma}(E_{\gamma}) <\xi_2(E_{\gamma})>$. From these equations one can find the energy and circular polarization distributions of γ -beam, if the dependencies n_0 and n_c are known. Obviously, similar equations may be written for $<\xi_1>$ and $<\xi_3>$ Stokes parameters of γ -beam. It should be noted that Eqs. (4)-(5) are valid when the intensity of laser wave is not very high (see [10, 11]).

The pair production process in single crystals [8, 9] may also be used for the determination of the energy spectrum and linear polarization of γ -beams. Generally, this process depends on the two angles of orientation relative to direction of γ -quanta propagation. When γ -beam move near strong crystallographic plane and far from axes, the process is described with the help of the one orientation angle θ_c with respect to the plane. Fig.1c illustrates the pair production cross sections for unpolarized γ -quanta propagating in a silicon single crystal near (110) plane. One can see that the cross sections are changed very strongly at variations of the orientation angle in the range 0.2 - 1 mrad. Unlikely, the cross section of pair production in single crystals depends on the linear polarization and is independent of the circular polarization [8, 9] (nevertheless, see [14]). It means that only $<\xi_1>$ and $<\xi_3>$ Stokes parameters may be determined with the help of the single crystals.

However, the circular polarization of γ -beam can be transformed into a linear one with the help of laser bunch [10, 11, 15], and thereafter can be analyzed by a single crystal. Note that this transformation take a place without intensity losses of γ -beam (if $E_{\gamma,max} < m^2c^4/E_l$).

Eq. (6) can be rewritten in the following differential form:

$$\frac{dn_i}{dE_e}(E_e, E_p) = n_2\delta x \int_{E_{\gamma,min}}^{E_{\gamma,max}} N_{\gamma}(E_{\gamma}) \frac{d\sigma_{\gamma\gamma}}{dE_e}(E_{\gamma}, E_e, E_p)dE_{\gamma}, \quad (9)$$

where $\frac{d\sigma_{\gamma\gamma}}{dE_e}$ is the differential cross section for production e^{\pm} pair when one of the particles has energy equal to E_e , E_p is a possible parameter such as an orientation angle, laser photon energy, etc. This equation is written for the fixed target ($v_2 = 0$). In principle, one can measure the dn_i/dE_e -value with the use of the magnetic analysis. Then Eq. (9) is a linear integral equation and its solution is the energy distribution of γ -beam. It is obvious, the measurement of the energy distribution of e^{\pm} -pairs allows one to use various media, for example, amorphous ones. Note, equations similar to Eqs. (7)-(8) one can write in differential form too.

Real γ -beams have some angular divergence. The constancy of the cross section $\sigma_{\gamma\gamma}$ in the limits of the divergence is necessary for using Eqs. (6)-(9). One can see from Eq. (4) that this condition are true for the laser medium, if the divergence less then tens mrad. In the case of the usage of silicon single crystals oriented near (110) plane the divergence $\delta\phi$ of the γ -beam must satisfy (only in one direction) the following condition: $\delta\phi/\theta_c \leq 0.02$.

3 Simulation

In this section we present some calculations, which can help to feel the problems of realization of energy and polarization measurements on the future $\gamma\gamma$ -colliders. For our calculations we select $E_{\gamma,max} = 400$ GeV. This energy corresponds to the electron beam energy about 500 GeV.

For a parallel and pointlike electron beam the number of scattered photons with the energy in the range from E_γ to $E_\gamma + dE_\gamma$ [1, 2] is

$$d\tilde{N}_\gamma = N_e \frac{k}{\sigma_{ce}} \frac{d\sigma_{ce}}{dE_\gamma} dE_\gamma \quad (10)$$

where N_e is the number of electrons, k is the conversion coefficient, σ_{ce} is the cross section of the Compton effect. For simulations we select γ -beam energy distribution in the form: $d\sigma_{ce}/dE_\gamma$. The form of the energy and polarization distributions depends on the helicities λ , P_c of electron and laser beams [2]. Fig.2 illustrates these distributions for two cases: 1) $2\lambda = 1$, $P_c = -1$; 2) $2\lambda = -1$, $P_c = -1$.

We calculate the n_0 and n_c -values (see Eqs. (7-8)) for the above-mentioned the energy and polarization distributions of γ -beam. Fig.3 illustrates these calculations in the laser wave and silicon single crystal. Note that n_c is equal to zero for single crystals. One can see that these functions are simple enough and have smooth shapes.

Knowing n_0 and n_c -functions one can make an attempt to find the energy and polarization distributions of γ -beam for the above-mentioned cases. The simplest way to do it is to represent of the unknown distributions in the form of polynomials

$$N_\gamma = \sum_{k=0}^N a_k E_\gamma^k, \quad (11)$$

$$N_\gamma < \xi_2 > = \sum_{k=0}^N b_k E_\gamma^k, \quad (12)$$

where a_k and b_k are the constant coefficients. Substitution of Eqs. (11)-(12) in Eqs. (7)-(8) allows to obtain the two systems of linear algebraic equations for different E_l -values. The solutions of these equations are a_k and b_k -coefficients.

We select $N=7$ for calculations. We also select the 8 various E_l -values in the follows manner: $E_l = E_{l,min} + \delta E_l k$, ($k = 0 - 7$). Here $E_{l,min}$ and δE_l are the minimal energy and step of energy variation for the laser photon. Some series of calculations were done, in which $E_{l,min}, \delta E_l$, are varied from 0.8 and 0.25 eV to 1.5 and 1 eV, correspondingly. The obtained in such a manner approximate polynomials are described good enough as the whole energy and polarization distribution for two cases. Fig.2 illustrates the typical description with the help of polynomials when $E_{l,min} = 1$ eV and $\delta E_l = 0.25$ eV. One can see that the narrow range near the end of energy and polarization distributions (from 390 to 400 GeV) are described incorrectly for the second case. However, if we select $E_{l,min} = 0.7$ eV and $\delta E_l = 0.05$ eV, then the polynomial description in a range from 300

to 400 GeV becomes good. Note, it is impossible to describe properly the origin of all distributions because of the threshold in γ -beam and laser photon interactions (see Fig.1). The agreement on fig.2 between the polynomial and true distributions at small energies one can explain by the simple forms of curves or by randomness. Similar calculations were also done with the use of the cross section in silicon single crystals (see Fig.2c) at variations of the orientation angle from 0.2 to 1 mrad. These calculations show good agreement between the computed and initial energy distributions.

We suppose that there is no problem to solve the integral equations numerically, if the n_0 and n_c -functions are known with a high precision. However, these functions will be obtained as result of measurements. Because of this, it is important to understand the influence on the reconstruction of the energy and polarization distributions such factors as fluctuations and systematic errors. Our estimations on the base of the polynomial model show that solutions of Eqs. (7)-(8) are stable enough relatively to small uniformly-distributed shifts of n_0, n_c -values (of about 1% to their absolute values). On the other hand, small (about 0.01%-0.001%) fluctuations of the above pointed values do not allow one to describe satisfactory the energy and polarization distributions with the help of polynomials at $N=7$. However, this result is obvious: fluctuations distort the true distributions and more high degree of polynomials or another class of approximation functions are required for their description. We think that for the discount of fluctuations it will be properly to correct the measured n_0 and n_c -values, for example, by the method of least squares.

4 Discussion

In this paper we have considered the possibility to measure the energy and polarization distributions of high energy γ -beam with intensity which does not allow to counter every γ -quantum. This situation is expected on future $e\gamma$ and $\gamma\gamma$ -colliders. One of main parameters of these colliders is the luminosity. The relations for spectral luminosity one can find in [1, 2]. In the general case, the knowledge of the energy and polarization distributions does not determinate completely the spectral luminosity and such important characteristics as the averaged $\langle \xi_i \xi_j \rangle$, ($i, j = 1 - 3$) Stokes parameters. However, we think that measurements of the above pointed distributions will be very useful for determination of the luminosity but further investigations in this field are required. Besides, in specific cases (but important for practice) the spectral luminosity is factored, and the energy and polarization distributions of both the γ -beams are multipliers in the product of some values. The spectral luminosity of $e\gamma$ and $\gamma\gamma$ -colliders will be determined by the use of the various calibration processes [1, 2]. Contrary to this practice the solution of the above pointed problem allows one to carry out the measurements for some hundreds cycles.

5 Conclusion

A new possibility for the measurements of the energy and polarization distributions of γ -beams in hundreds GeV is proposed. This possibility may be useful for practical applications on future $e\gamma$ and $\gamma\gamma$ -colliders. However, further development of this possibility is required. In particular, it is necessary to find solutions of the following problems:

- i) the choice of optimal algorithm for solution of Eqs. (7)-(9);

ii) the reconstruction of spectral luminosity on the base of measurements of energy distributions of γ -beams;

iii) the choice of the detector type for registration of e^\pm -pairs;

iv) the estimation of the background and its minimization.

At the present time the considered method may be investigated experimentally on γ -beams of electron accelerators [16].

We believe that another important problem may be solved on the base of our consideration. This is the problem of experimental determination of the equivalent photon spectrum in peripheral collisions of relativistic ions [17].

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References

- [1] I. Ginzburg, G. Kotkin, V. Serbo, V. Telnov, Nucl. Instr. Meth, **205** (1983) 47.
- [2] I. Ginzburg, G. Kotkin, S. Panfil, V. Serbo, V. Telnov, Nucl. Instr. Meth. **219** (1984) 5.
- [3] K. Abe et al. hep-ph/0109166.
- [4] B. Badelek et. al. TESLA TDR, Part IV: The Photon Collider at TESLA, hep-ex/0108012.
- [5] JLC Design Study, KEK-REP-97-1, April 1997. I. Watanabe et. al. KEK Report 97-17.
- [6] V.B. Berestetskii, E.M. Lifshits, and L.P. Pitaevskii, Quantum Electrodynamics, Pergamon, Oxford (1982).
- [7] A.I. Nikishov, V.I. Ritus Zh. Eksp. Teor. Fiz., **52** (1967) 1707.
- [8] H.Uberall, Phys.Rev., **107** (1957) 223.
- [9] M.A.Ter-Mikaelyan, High Energy Electromagnetic Processes in Condensed Media, Wiley, New York, 1972.
- [10] G.L. Kotkin, V.G. Serbo, Phys. Lett. **B 413** (1997) 122.
- [11] V.A. Maisheev, Zh. Eksp. Teor. Fiz. **112** (1997) 2016 [in Russian]; JETP **85** (1997) 1102. Program for PC (cvt.exe) is available on server:
<http://dbserv.ihep.su~pubs/prep1997/97-25-e.htm>
- [12] V.A. Maisheev, Nucl. Instr. Meth. **B 168** (2000) 11.
- [13] L.D. Landau, and E.M. Lifshitz, The Classical Theory of Fields, Pergamon, Oxford (1975).
- [14] V.A. Maisheev Phys. Lett. **B 477** (2000) 83.
- [15] V.A. Maisheev, hep-ph/0101191.
- [16] V. Ghazikhanian et. al. SLAC-PROPOSAL E-159, 2000.
- [17] G. Baur et. al, hep-ph/0112211.

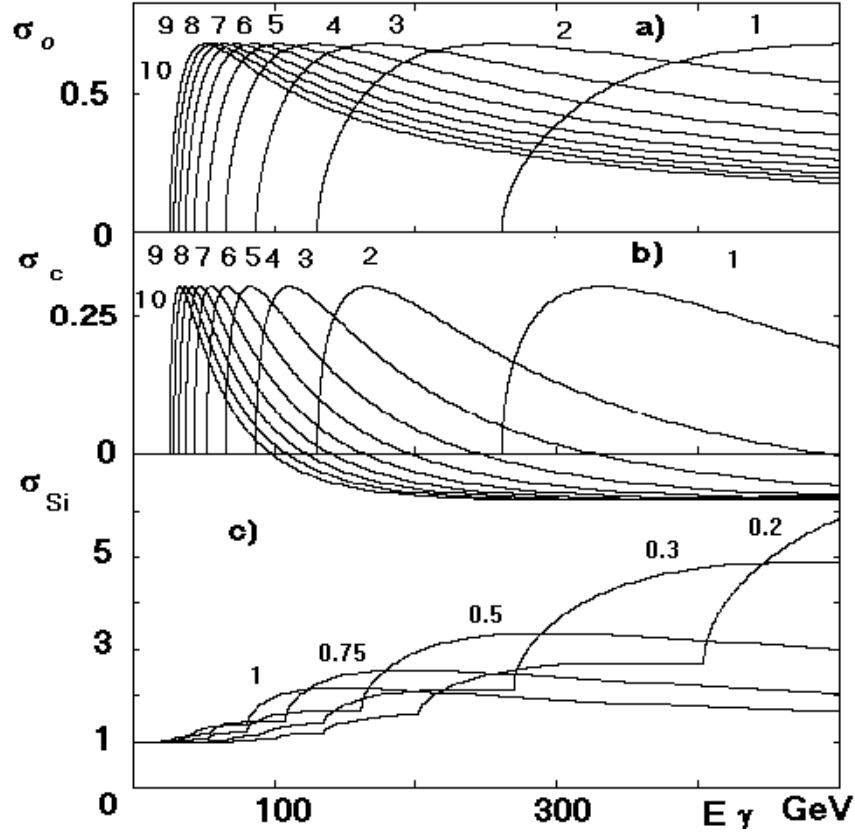


Figure 1: The cross sections of e^\pm -pair production σ_0 , σ_c in laser wave (a,b) and σ_{Si} in (110) silicon single crystal plane (c) as functions of γ -quantum energy. The numbers near curves are the energy of laser photon (a,b) in eV and orientation angle (c) in mrad. σ_0 and σ_c is measured in πr_e^2 units, where r_e is electron radius. σ_{Si} is normalized on the cross section when the crystal is not aligned.

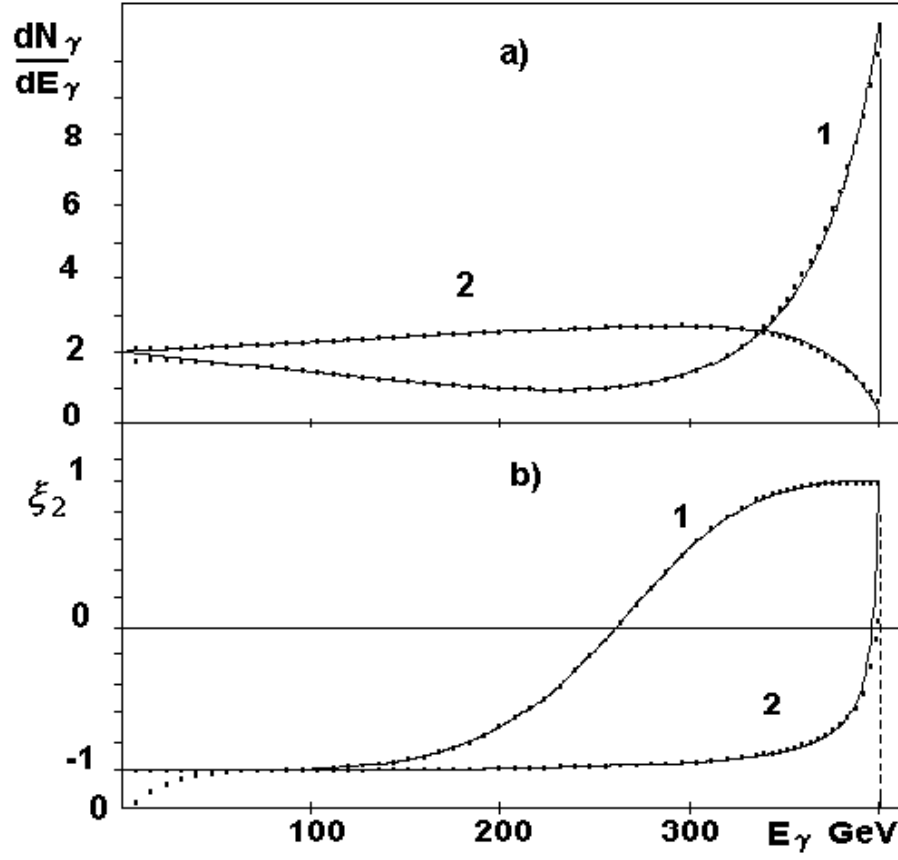


Figure 2: The energy (a) and circular polarization (b) distributions of γ -beam for two cases: 1) $2\lambda=1$, $P_c=-1$; 2) $2\lambda=-1$, $P_c=-1$. The pointed curves are result of reconstruction (see text for detail).

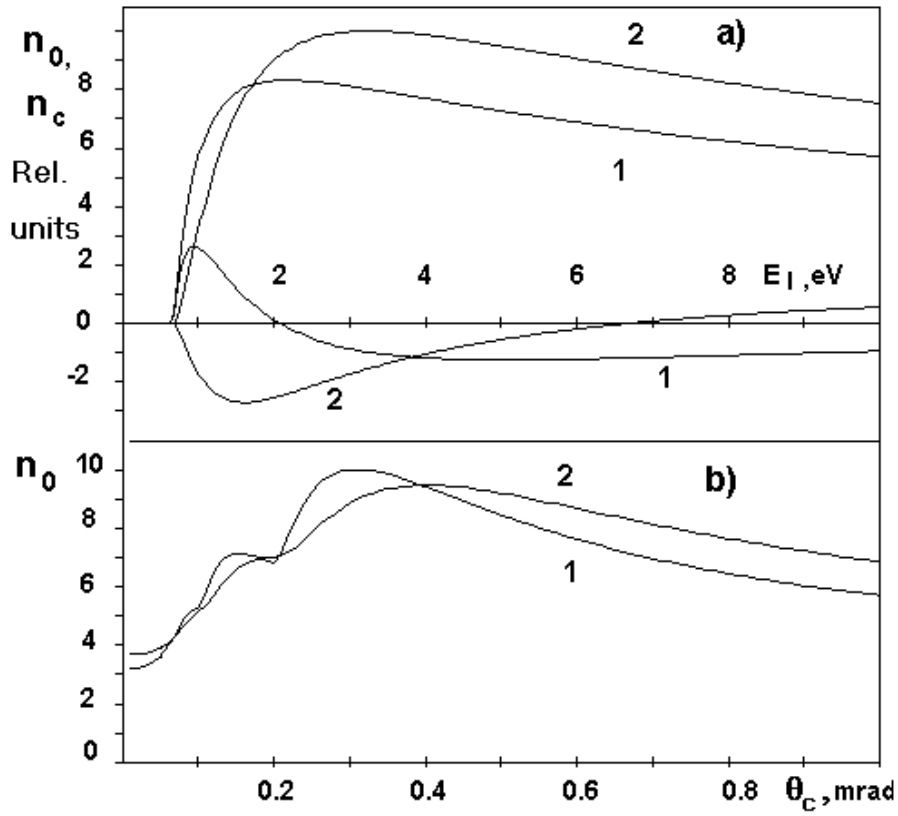


Figure 3: a) Calculated n_0 and n_c -values as functions of laser photon energy; b) calculated n_0 -values as functions of the orientation angle of silicon single crystal relative to (110) plane. Numbers near curve correspond to: 1) $2\lambda=1, P_c=-1$; 2) $2\lambda=-1, P_c=-1$.